Meriam 3/8, 3/9: Linear Impulse & Momentum

For a particle of constant mass m,

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$\rightarrow \quad \int_{t_1}^{t_2} \sum \vec{F}\, dt = \int_{\vec{V}_1}^{\vec{V}_2} m\, d\vec{v}$$

$$= m\vec{V}_2 - m\vec{V}_1$$

$$\rightarrow \quad m\vec{v}_2 = m\vec{v}_1 + \int_{t_1}^{t_2} \vec{F}\, dt$$

or $$\vec{G}_2 = \vec{G}_1 + \int_{t_1}^{t_2} \vec{F}\, dt$$

where $\vec{G} = m\vec{v}$ = linear momentum (vector)

and $\int_{t_1}^{t_2} \vec{F}\, dt$ = linear impulse (vector)

$= \vec{F}_{average}\, \Delta t$

Compare:

$$\vec{G}_2 = \vec{G}_1 + \int_{t_1}^{t_2} \vec{F}\, dt$$

$$T_2 = T_1 + \int_S \vec{F} \cdot d\vec{r}$$

Eg:

Consider a 100-kg man in a car moving at 20 m/s $\hat{i}$. The car crashes into a wall; the man's speed goes rapidly to 0.

$$\vec{G}_2 = \vec{G}_1 + \vec{F}_{average}\,\Delta t$$

$\vec{G}_2$ → 0

$\vec{G}_1$ → $(100)(20) = 2000 \frac{kg \cdot m}{s}$

The car manufacturer cannot control $\vec{G}_1$, $\vec{G}_2$, or the total linear impulse required to stop the man.

$\vec{F}_{average}$ (N)	×	Δt (s)	=	−2000 kg·m/s $\hat{i}$
$-100\ \hat{i}$		20		$-2000\ \hat{i}$
$-500\ \hat{i}$		4		$-2000\ \hat{i}$
$-2{,}000\ \hat{i}$		1		$-2000\ \hat{i}$
⋮				

Consider kinesiology of human body

Note that with a single mass one can solve Linear Momentum problems just using $\vec{F} = m\vec{a}$

The concept of Linear Momentum becomes essential when one deals with collisions: m_1 & m_2

15(c)

LAST Name: Answers

First Name: _____

Student Number: _____

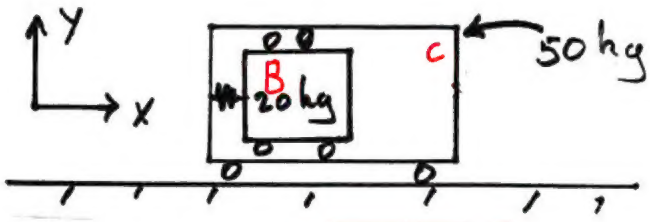
MIE 200F - Quiz number 5b – October 18, 2001
quiz duration = 20 minutes

A hollow crate of mass $m_C = 50$ kg is mounted on frictionless wheels. Inside the crate sits a box of mass $m_B = 20$ kg mounted on frictionless wheels. At time $t = 0$, the system is released from rest, with the spring having an initial length of 0.4 meters. The spring constant is $k = 30$ N/m, and the spring has a relaxed length of 2.5 meters.

(a) What is the initial potential energy of the spring?
(b) What will be the initial acceleration of the box?
(c) What will be the speed of the crate when the spring reaches a length of 1.5 meters?

$V_e = \frac{1}{2} kx^2$ $\quad V_{gravity} = mgh$ $\quad U_{gravity} = -mg\Delta h$ $\quad U_{spring} = \frac{1}{2} k (x_1^2 - x_2^2)$ $\quad \vec{F}_{spring} = -kx$

$T_2 = T_1 + U_{1\to 2}$ $\quad \Sigma \vec{M}_o = \vec{H}_o$ $\quad \vec{G} = m\vec{v}$ $\quad \vec{H}_o = \vec{r} \times m\vec{v}$ $\quad T = \frac{1}{2} mv^2$ $\quad \Delta\vec{H}_o = \int \vec{M}_o \, dt$



(a) $V_e = \frac{1}{2} k x^2 = \left(\frac{1}{2}\right)(30)(.4 - 2.5)^2 =$ **66.15 J**

(b) $\vec{F} = m\vec{a}$: $\quad -(30)(-2.1\,\hat{i}) = (20\text{ kg})(\vec{a})$

$\vec{a} = 3.15\,\hat{i}$ m/s²

(c) $T_2 = \cancel{T_1}^{\,0} + U_{1\to 2}$ $\quad \left(\frac{1}{2}\right)(50) V_C^2 + \left(\frac{1}{2}\right)(20)(V_B)^2 = -\frac{1}{2} k \left(x_2^2 - x_1^2\right)$

$= -\left(\frac{1}{2}\right)(30\text{ N/m})\left((-1)^2 - (-2.1)^2\right)$ (*)

$25 V_C^2 + 10 V_B^2 = 51.15$

Also, $\Delta G_x = 0 \Rightarrow (G_2)_x = (G_1)_x = 0 \Rightarrow$ $\quad 50 V_C + 20 V_B = 0$

$\Rightarrow$ **$V_B = -2.5 V_C$** (**)

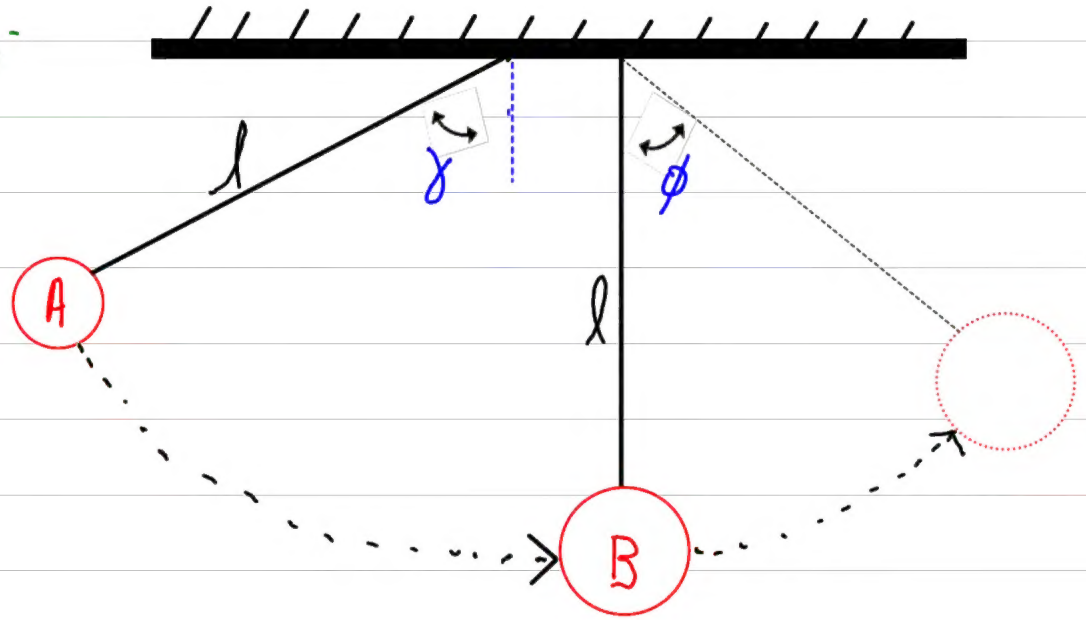
combine (*) & (**)

$25 V_C^2 + 10(-2.5 V_C)^2 = 51.5$

$V_C = \pm 0.767$ m/s.

From diagram, **$V_C = -0.767\,\hat{i}$ m/s.**

Eg.



Find rebound angle ϕ of ball B if ball A is released from rest at angle γ, & stops after collision.

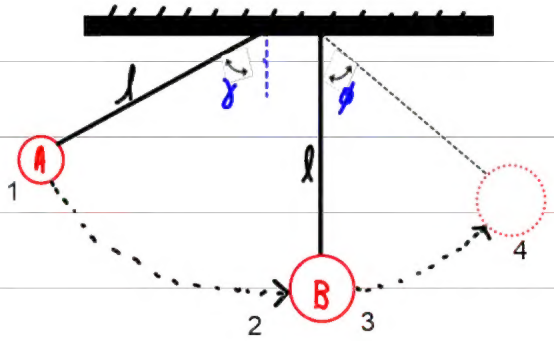
Given:

$m_A = 1.5$ kg

$m_B = 3.0$ kg

$\gamma = 70°$

$V_A = 0$ after impact



Define 4 milestones in this trip:

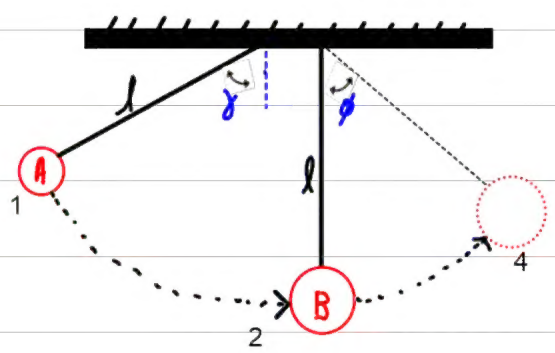
1 = initial
2 = pre-impact
3 = post-impact
4 = maximum rebound

journey (a): energy eqn
journey (b): momentum eqn
journey (c): energy eqn

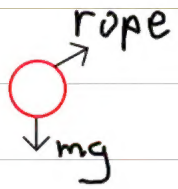
→ We divide total trip into 3 separate journeys, and solve them in sequence.

15 (e)

(a) position 1 $\longrightarrow$ position 2



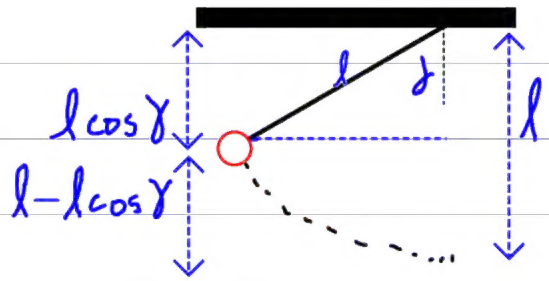
$T_1 + V_1 = T_2 + V_2$



or $T_2 = T_1 + U_{1\to 2}$ $\quad -mg\Delta h$

mass A: $\frac{1}{2} m_A v_2^2 = 0 - (m_A)(g)\left[-(l - l\cos\gamma)\right]$ $\quad \gamma = 70°$

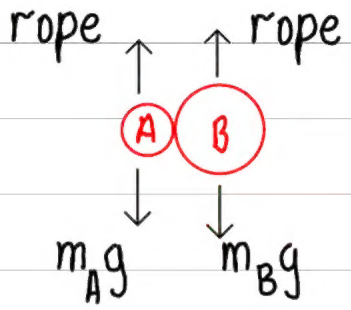
mass A: $v_2 = 1.147\sqrt{gl}$



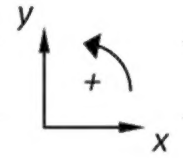
(b) position 2 $\longrightarrow$ position 3

Consider the (A) (B) system

No external forces in x-direction



$\rightarrow (G_2)_x = (G_3)_x$

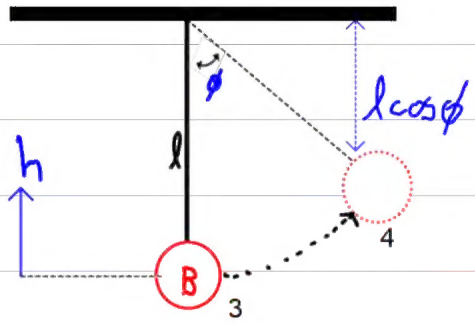


$m_A (v_A)_2 + m_B (v_B)_2 = m_A (v_A)_3 + m_B (v_B)_3$

1.5 $\quad 1.147\sqrt{gl}$ $\quad 0$ $\quad 0$ $\quad 3$

$\rightarrow$ **$(v_B)_3 = 0.574\sqrt{gl}$**

(c) position 3 $\longrightarrow$ position 4



$$T_3 + V_3 = T_4 + V_4$$

$$\tfrac{1}{2} m_B (v_B)_3^2 + 0 = 0 + (m_B)(g)[l - l\cos\phi]$$

$0.574\sqrt{gl}$ $\rightarrow$ $\cos\phi = 0.835$

$$\phi = 33.3^\circ$$

Summary

- $\vec{G} = m\vec{v}$ = linear momentum (vector)
- $\vec{G}_2 = \vec{G}_1 + \int_{t_1}^{t_2} \vec{F}\, dt$
- $\vec{G}$ is useful for modeling collisions, explosions

For Next Lecture

- Read Mesiam 3/10 : Angular momentum